MATH 140A Review: Quantifiers and Negation

Facts to Know:

Quantifiers:

- (**For all** ∀) The statement holds for all x.

- (**There exists** ∃) The statement holds for some x.

Negation:

- (*not* **for all**) The statement is false for some x.

- (*not* **there exists**) The statement is false for every x.

When negating a statement with quantifiers, remember to

- First, the statement that needs to hold.

- If there is more than one quantifier, handle them one at a time from left to right.

Examples:

1. Negate the following statement: <u>Every</u> human has an X-chromosome.

   ↓

   There exists a human that does not have an X-chromosome.

2. Negate the following statement: <u>There exists</u> a human with a Y-chromosome.

   ∃

   Every human does not have a Y-chromosome.

3. Negate the following statement: Let a_n be a sequence of real numbers and let L be a real number. For every $\epsilon > 0$, there exists $N \in \mathbb{N}$ such that for all $n \in \mathbb{N}$, if $n \geq N$, then $|a_n - L| < \epsilon$.

$$\forall \, \epsilon > 0, \; \exists N \in \mathbb{N}, \; \forall n \in \mathbb{N}, \; \text{if } n \geq N, \text{ then } |a_n - L| < \epsilon$$

$$\exists \epsilon > 0, \; \forall N \in \mathbb{N}, \; \exists n \in \mathbb{N}, \; n \geq N \text{ and } |a_n - L| \geq \epsilon.$$

There exists $\epsilon > 0$ such that for all $N \in \mathbb{N}$, there exists $n \in \mathbb{N}$ such that

$$n \geq N \quad \text{and} \quad |a_n - L| \geq \epsilon.$$